GATE CRASH COURSE (ALL BRANCHES)

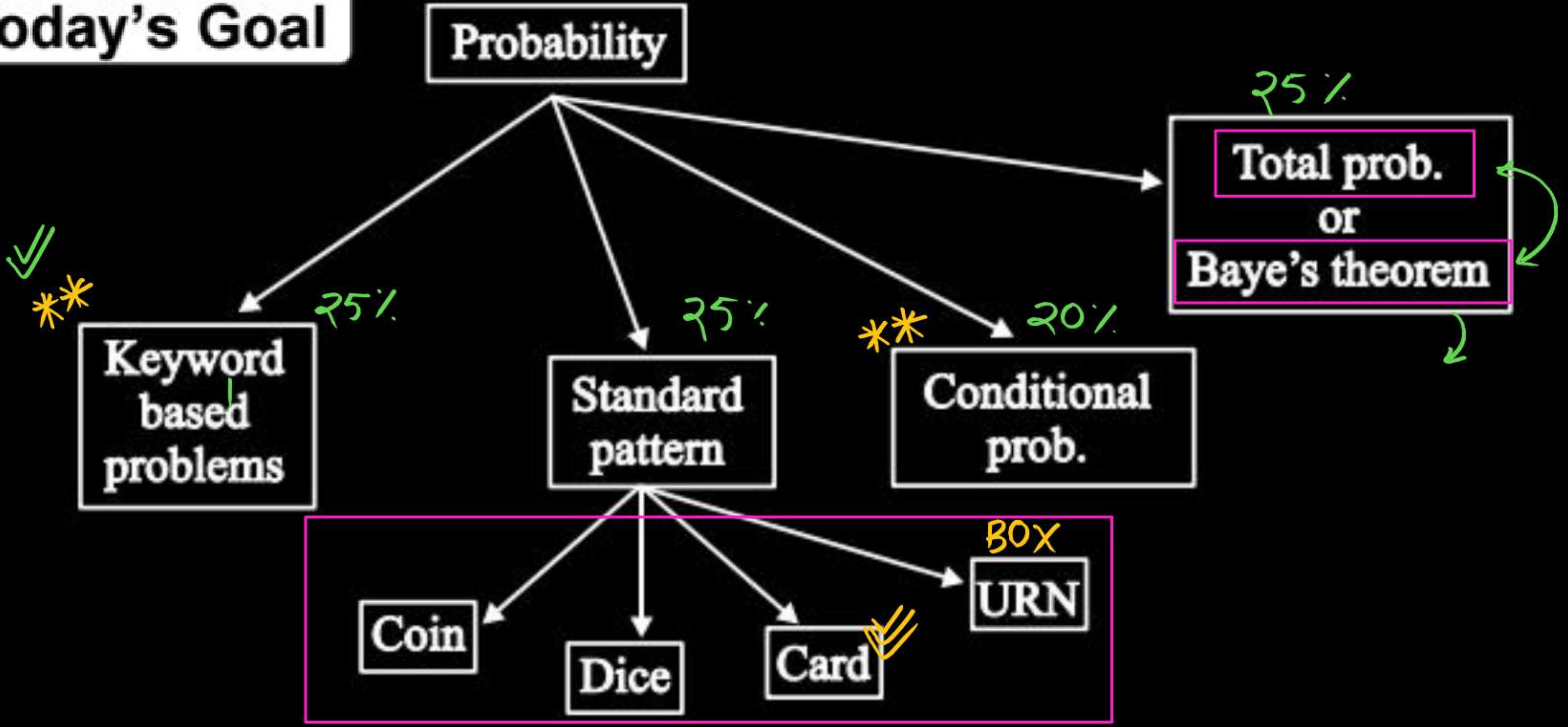
Engineering Mathematics

Probability

LECTURE NO.31

Vishal Soni Sir

Today's Goal

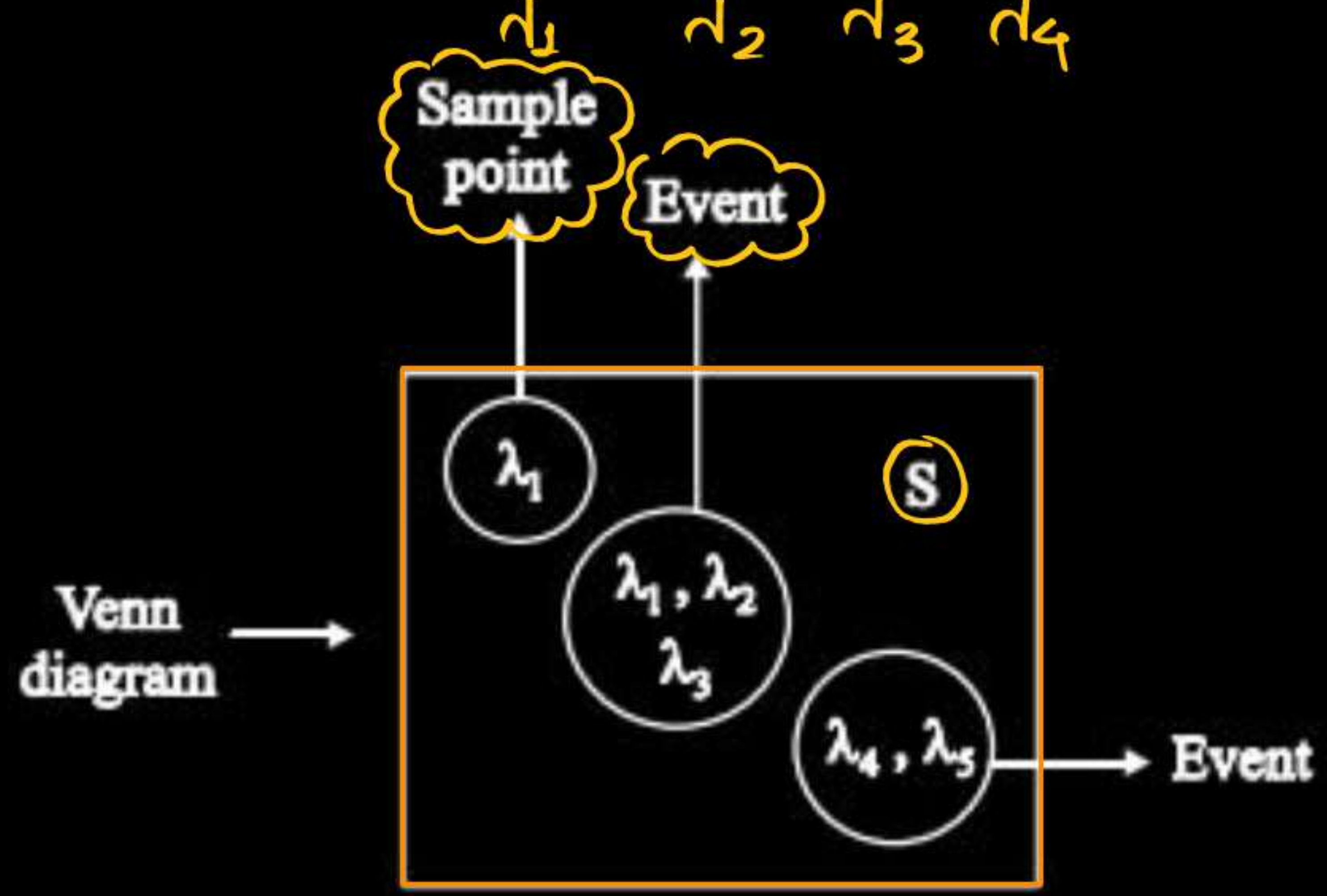


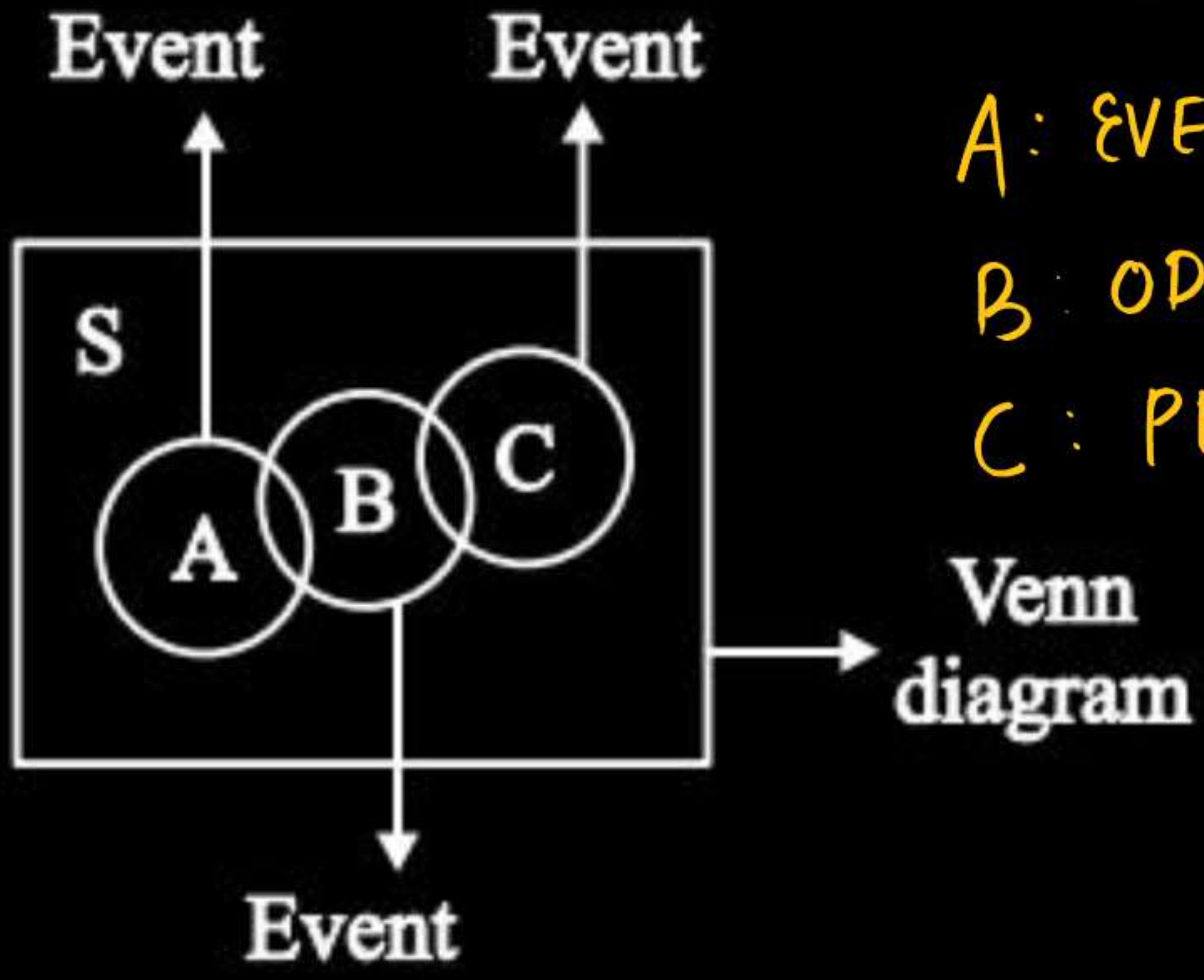
Some different variation

General Venn diagram representation

→ R.E. : 2 coins are tossed simultaneously

→ Sample space : $S = \{TT, TH, HT, HH\}$





$S = \{1, 2, 3, 4, 5, 6\}$

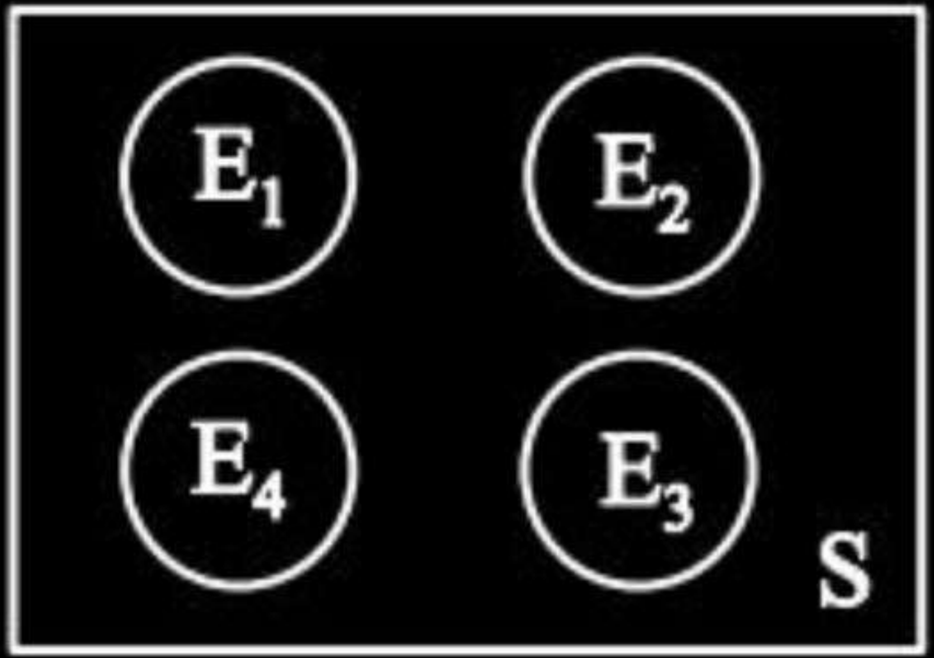
A: EVEN FACES $= \{2, 4, 6\}$

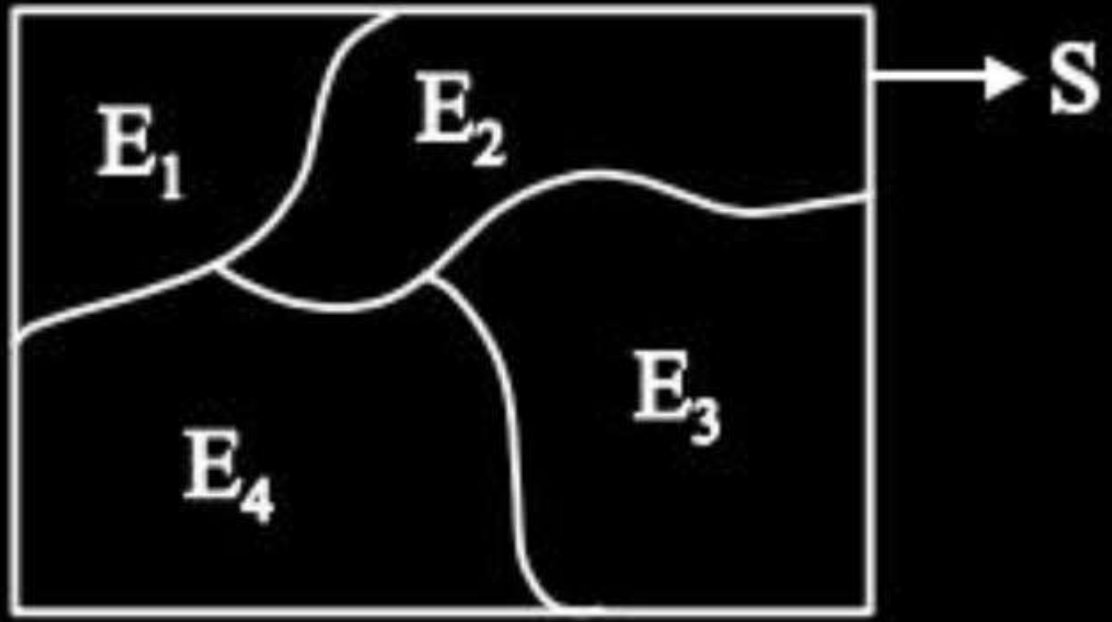
B: ODD FACES $= \{1, 3, 5\}$

C: PRIME FACES $= \{2, 3, 5\}$

$P(s) = 1$

$0 < P(A) < 1$, $0 < P(B) < 1$, $0 < P(C) < 1$





$P(s) = 1$

$0 < P(E_i) < 1$ $\quad i = 1, 2, 3, 4$

$0 \le P(E_1) + P(E_2) + P(E_3) + P(E_4) \le 1$

$P(s) = 1$

$0 < P(E_i) < 1$ $\quad i = 1, 2, 3, 4$

$P(E_1) + P(E_2) + P(E_3) + P(E_4) = 1$

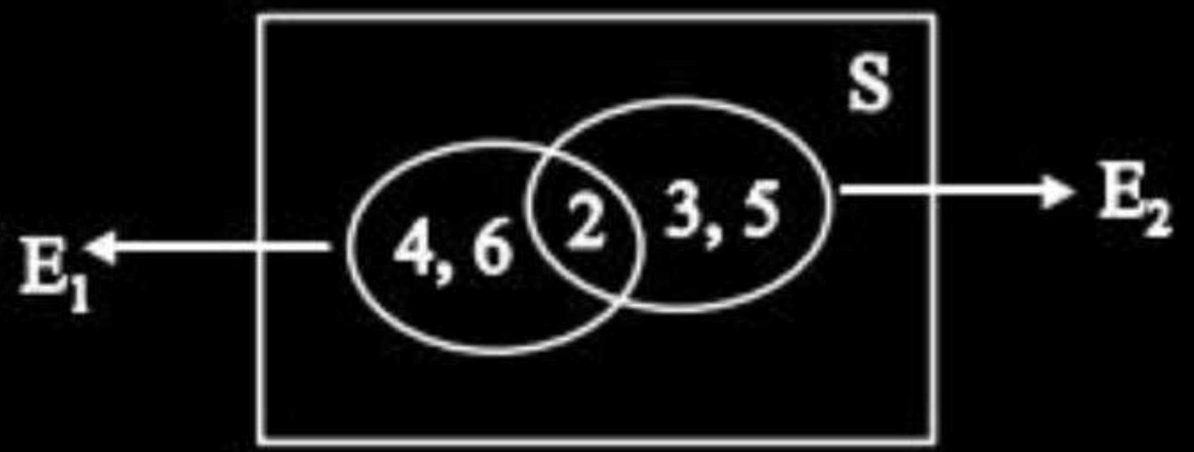
Ex: A dice is rolled

$S = \{1, 2, 3, 4, 5, 6\}$

E_1 : Outcomes should have even faces

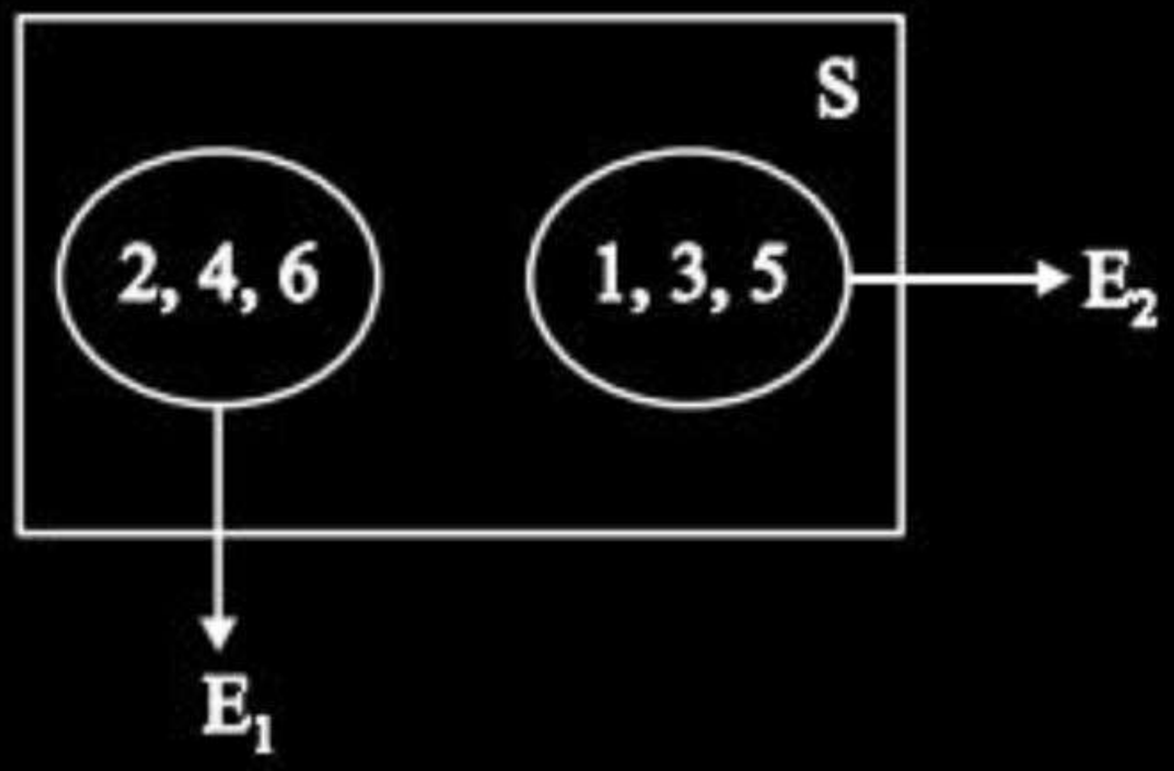
E_2 : Outcomes should have prime faces

$E_1 = \{2, 4, 6\}$ $E_2 = \{2, 3, 5\}$



E_1 : Outcome should have "Even" faces $= \{2, 4, 6\}$

E_2 : Outcome should have "Odd" faces $= \{1, 3, 5\}$

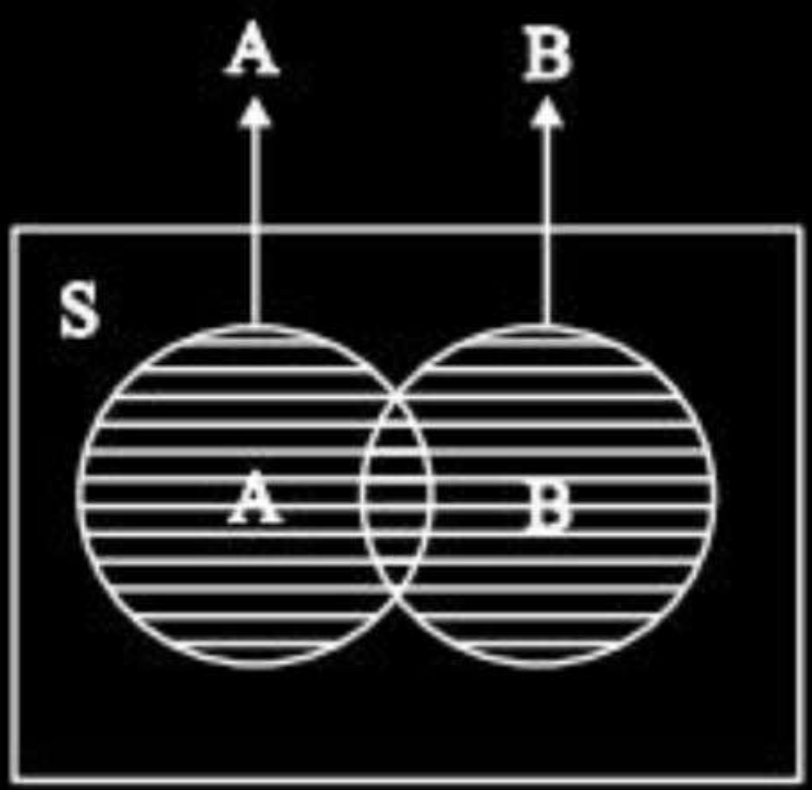


Union of events :

$A \cup B$

$A = \{1, 2, 3\}$

$B = \{2, 3, 5, 7\}$



$A \cup B = \{1, 2, 3, 5, 7\}$ = Either A or B or both

Imp Points:

1. When the set A and B are two events associated with sample space S

$A \cup B$ is an event $\rightarrow$ "Either A or B or both"

$\rightarrow$ "At least one of A or B will occur."

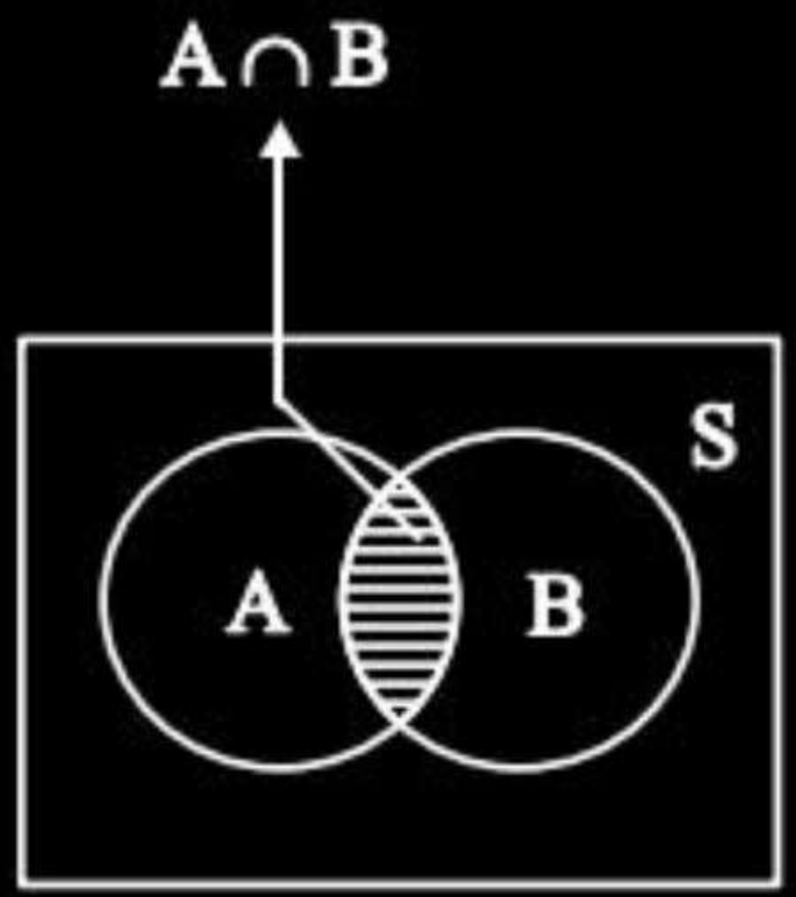
Intersection of events:

$A \cap B$

$A = \{1, 2, 3\}$

$B = \{2, 3, 4, 5, 7\}$

$A \cap B = \{2, 3\}$



Imp point : When set A and B are two events associated with sample space S

$A \cap B$: Event → A and B will occur simultaneous

→ Both A and B will occur

Case 1:

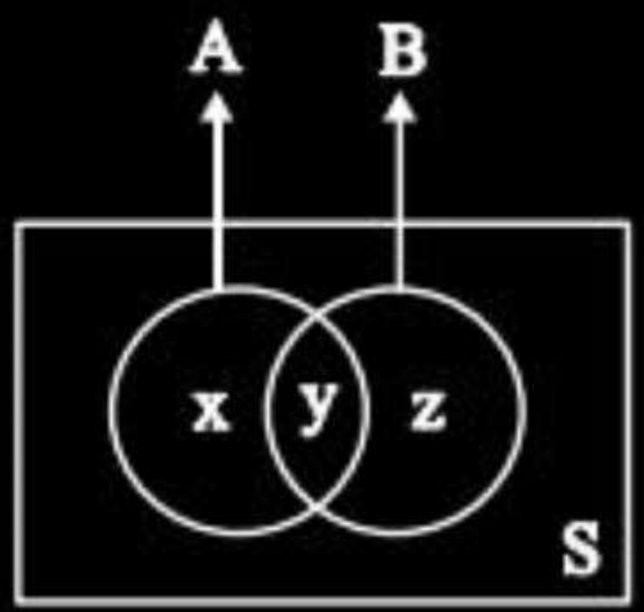
Atleast one of A or B should occur

or

Either A or B or both should occur

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$ **

Method 1 :



$n(A) = x + y$

$n(B) = y + z$

$n(A \cap B) = y$

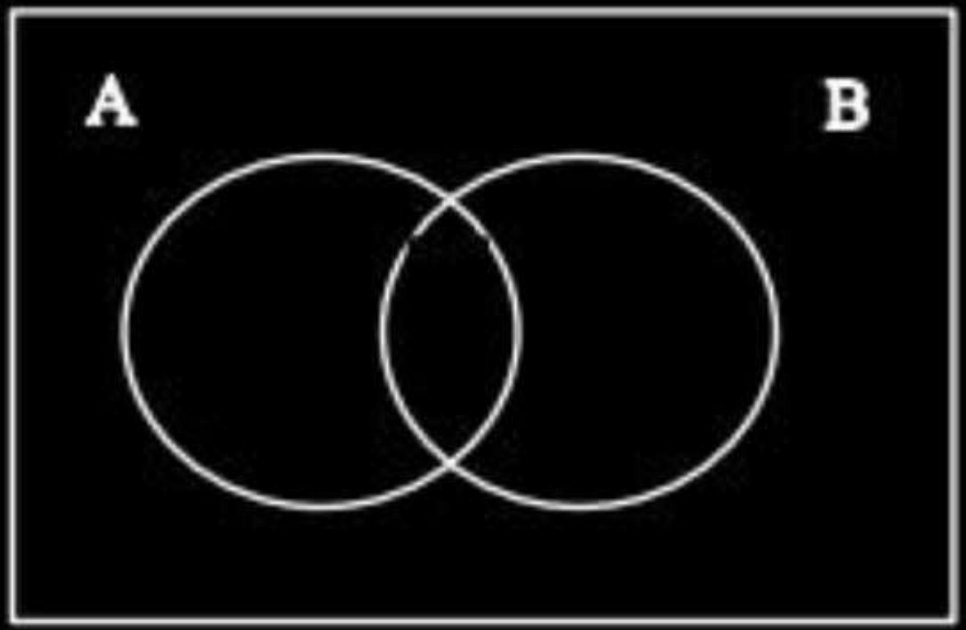
$n(A \cup B) = x + y + z$

$$\frac{n(A \cup B)}{n(s)} = \frac{n(A)}{n(s)} + \frac{n(B)}{n(s)} - \frac{n(A \cap B)}{n(s)}$$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Method 2:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

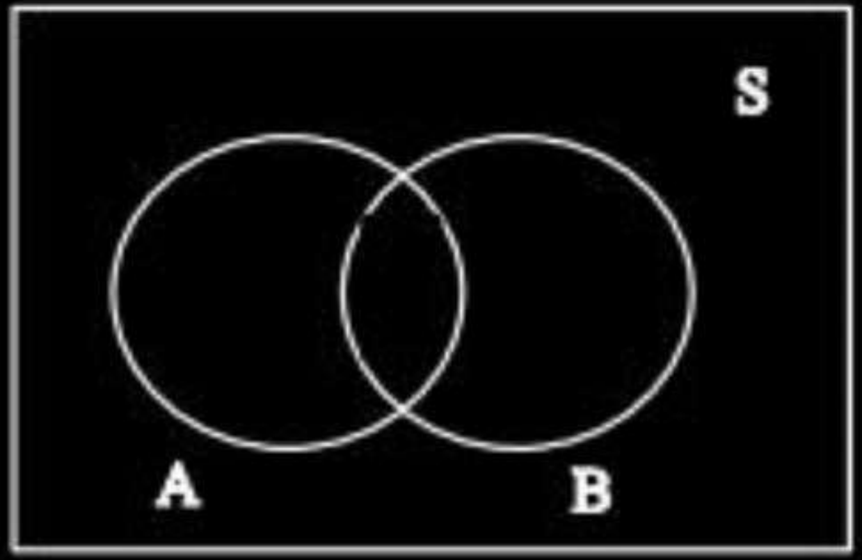


Case 2 :

Exactly one of A or B occurs = E

$$P(E) = P(A) + P(B) - 2P(A \cap B)$$

$$P(E) = P(A \cup B) - P(A \cap B)$$



Case 3:

"Neither A nor B."

P (Neither A nor B) = {1 – P(Either A or B)}

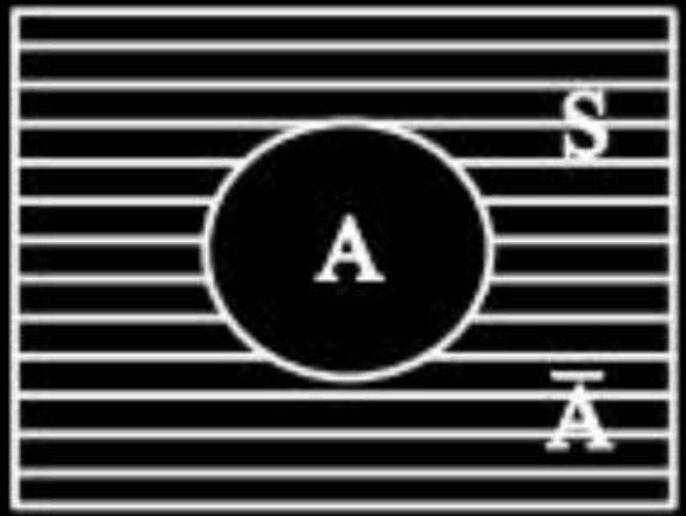
= {1- P(Either A or B or both)}

$$P(\bar{A} \cap \bar{B}) = P(\text{neither A nor B}) = 1 - P(A \cup B)$$

$$\overline{P(A \cup B)} = 1 - P(A \cup B)$$

Case 4:

Compliment of an event:



$E_1 : A$

$E_2 : \bar{A}$

$P(A) + P(\bar{A}) = 1$

$P(E_1) + P(E_2) = 1$

Summary :

For two events A and B of a sample space

(1) P (at least one of A or B occurs) =

or

P(Either A or B or both occurs) =

$$P(E) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(2) Exactly one of A or B occurs

$$P(E) = P(A) + P(B) - 2P(A \cap B) = P(A \cup B) - P(A \cap B)$$

(3) Neither A nor B occurs

$$P(E) = 1 - P(A \cup B)$$

Case 5:

Let A, B, C are 3 events of a sample space

At least one of A or B or C should occur

**

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Case 6:

Exactly two of the event A, B, C occurs:

$$P(E) = P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$$

EXACTLY 2 EVENT OUT OF A, B, C

$$P(A\cap B)+P(B\cap C)+P(C\cap A)-3P(A\cap B\cap C)$$

EXACTLY 1 EVENT OUT OF A, B, C

$$P(A)+P(B)+P(C)-2P(A\cap B)-2P(B\cap C)-2P(C\cap A)+3P(A\cap B\cap C)$$

Case 7.

Exactly one of the events of A, B, C should take place

$$P(E) = P(A) + P(B) + P(C)$$
$$-2P(A \cap B) - 2P(A \cap C)$$
$$-2P(B \cap C) + 3P(A \cap B \cap C)$$

Summary :

A, B, C are 3 events of a sample space:

(1) At least one of A, B, C should occur:

$$P(E) = P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

(2) Exactly 2 of A, B, C should occur:

$$P(E) = P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$$

(3) Exactly 1 of A, B, C should occur:

$$P(E) = P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(B \cap C) - 2P(C \cap A) + 3P(A \cap B \cap C)$$

Conditional probability

Let A and B are 2 events associated with same sample space. The conditional probability of an event A given that event B has already occurred.

*

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \text{ where } P(B) \neq 0$$

$$P(A \cap B) = P(A)\,P\left(\frac{B}{A}\right) = P(B)\,P\left(\frac{A}{B}\right)$$

(I) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(B)} : P(B) \neq 0$

(II) $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)} : P(A) \neq 0$

(III) $P(A \cap B) = P(B \cap A) = P(A)P\left(\frac{B}{A}\right) = P(B)P\left(\frac{A}{B}\right)$

Q. A and B are event such that

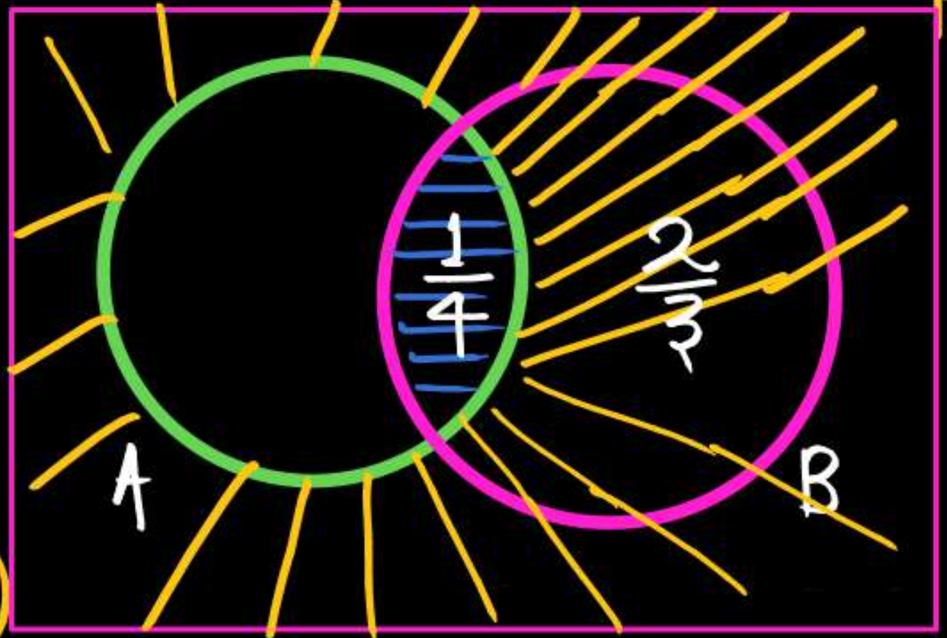
$P(A \cup B) = 3/4$ $P(A \cap B) = 1/4$

$P(\bar{A}) = 2/3 \longrightarrow P(A) = \frac{1}{3}$

$P(\bar{A} \cap B) = ?$

ONLY B No A

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

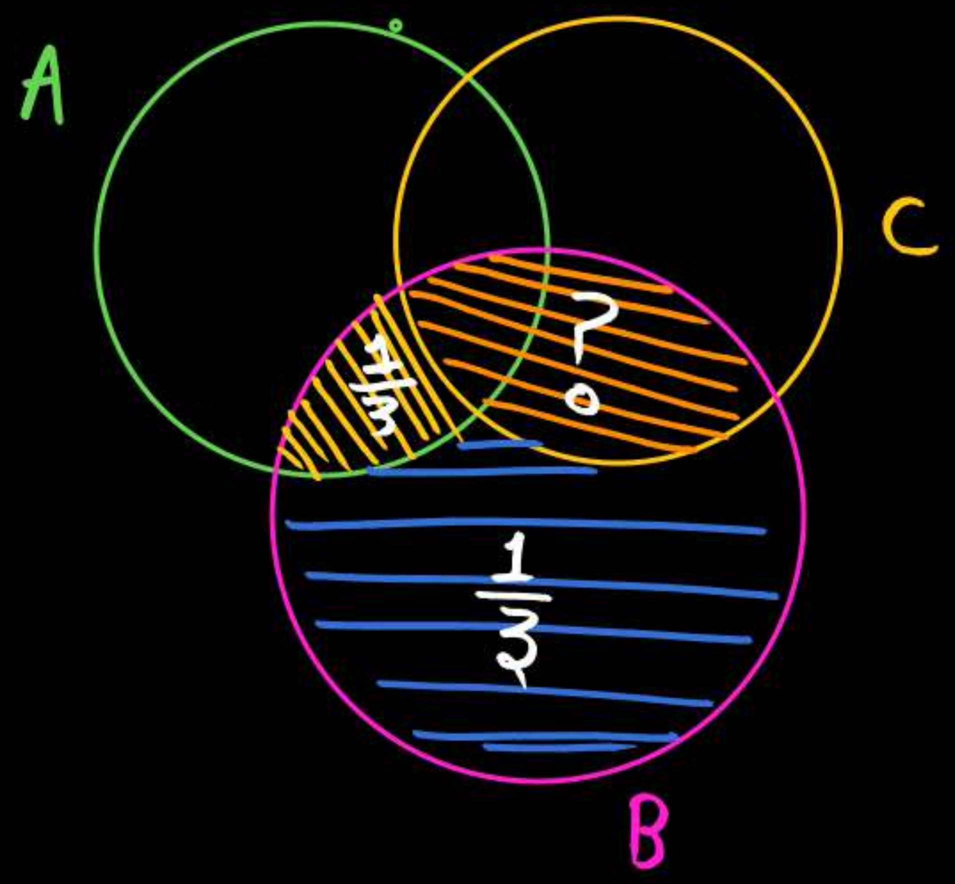
$\frac{3}{4} = \frac{1}{3} + P(B) - \frac{1}{4}$

$P(B) = \frac{3}{4} - \frac{1}{3} + \frac{1}{4} = \frac{2}{3}$

$= P(B) - P(A \cap B) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

Q. $P(B) = 3/4, \quad P(A \cap B \cap \bar{C}) = 1/3$

$P(\bar{A} \cap B \cap \bar{C}) = 1/3 \qquad P(B \cap C) = ?$



$$P(B \cap C) = P(B) - P(A \cap B \cap \bar{C}) - P(\bar{A} \cap B \cap \bar{C})$$

$$= \frac{3}{4} - \frac{1}{3} - \frac{1}{3}$$

$$= 1/12$$

Q. For 3 event A, B and C

P (Exactly one of A or B occurs) = P (Exactly one of B or C occurs) = P (Exactly one of A or C occurs) = 1/4

P (all the events occur simultaneously) = 1/16 $= P(A \cap B \cap C)$

P (at least one of the events occurs) = ? $P(A \cup B \cup C) = ?$

$$P(A)+P(B)-2P(A\cap B)=\frac{1}{4} \quad \text{—— (i)}$$

$$P(B)+P(C)-2P(B\cap C)=\frac{1}{4} \quad \text{—— (ii)}$$

$$P(C)+P(A)-2P(C\cap A)=\frac{1}{4} \quad \text{—— (iii)}$$

$$P(A\cap B\cap C)=1/16$$

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$$

$$\frac{3}{8}+\frac{1}{16}=\frac{7}{16}$$

$$2\{P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)\}=\frac{3}{4}$$

Q. Events A, B, C are mutually exclusive events such that

(i) ∩ (ii) ∩ (iii) ∩ (iv)

$$\frac{1}{3} \le x \le \frac{1}{2}$$

$$0 \le \frac{3x+1}{3} \le 1 \quad \text{— (i)}$$

$$0 \le \frac{1-x}{4} \le 1 \quad \text{— (ii)}$$

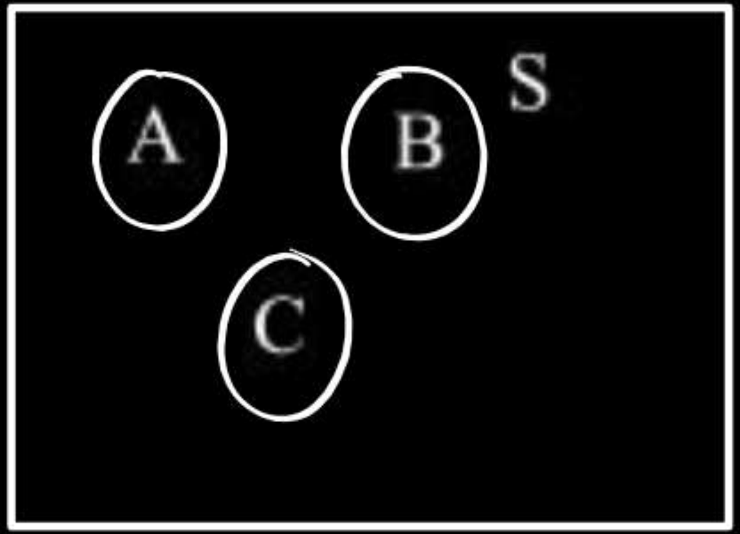
$$0 \le \frac{1-2x}{2} \le 1 \quad \text{— (iii)}$$

$$0 \le \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1 \quad \text{— (iv)}$$

$$P(A) = \frac{3x+1}{3}$$

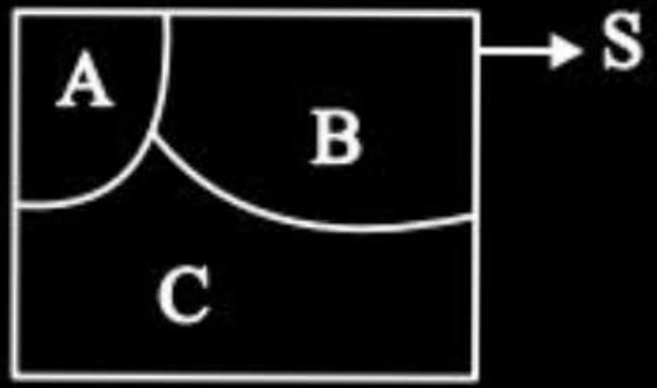
$$P(B) = \frac{1-x}{4}$$

$$P(C) = \frac{1-2x}{2}$$



The set of positive value of x are in the interval = ?

Q. Events A, B, C are mutually exclusive and exhaustive events such that



$$P(A) = \frac{3x+1}{3}$$

$$P(B) = \frac{1-x}{4}$$

$$P(C) = \frac{1-2x}{2}$$

$0 \leq \frac{3x+1}{3} \leq 1$ — (i)

$0 \leq \frac{1-x}{4} \leq 1$ — (ii)

$0 \leq \frac{1-2x}{2} \leq 1$ — (iii)

$$\frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} = 1$$

$x = 1/3$

$x = \frac{1}{3}$

The set of positive value of x are in the interval = ?

Types of events :

(1) Equally likely events :-

"Events are equally likely if they have same probability of occurrence."

Ex : Rolling of a dice

$S = \{1, 2, 3, 4, 5, 6\}$

E_1 : Outcomes are having off faces $E_1 = \{1, 3, 5\}$

E_2 : Outcomes are having prime faces $E_2 = \{2, 3, 5\}$

$$P(E_1) = \frac{n(E_1)}{n(s)} = \frac{3}{6} = \frac{1}{2}$$

$$P(E_2) = \frac{n(E_2)}{n(s)} = \frac{3}{6} = \frac{1}{2}$$

$P(E_1) = P(E_2)$

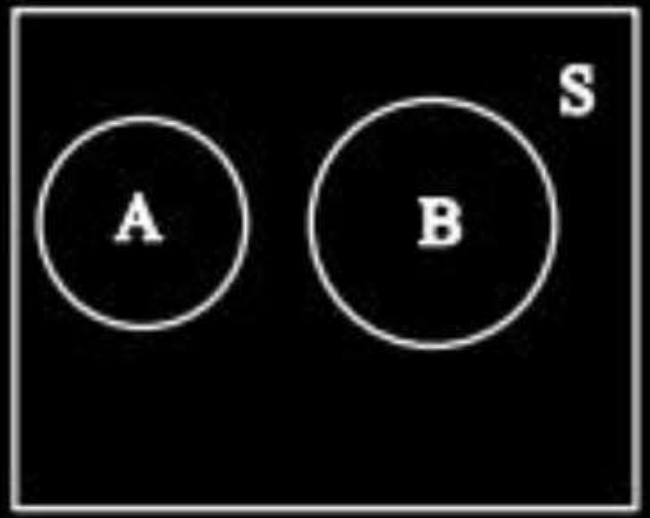
E_1 and E_2 are equally likely event

(2) Mutually exclusive or Disjoint events

Imp

Case 1:

Two event A and B are mutually exclusive if they can not occur together where A and B are events from same sample space



$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B)$$

If A and B are mutually exclusive events :

(1) $P(A \cap B) = 0$

(2) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = 0$

(3) $P\left(\frac{B}{A}\right) = 0$

(4) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B)$

Ex. Rolling of a dice

$S = \{1, 2, 3, 4, 5, 6\}$

E_1 = outcome is even faces = $\{2, 4, 6\}$ → $P(E_1) = 1/2$

E_2 = outcome is odd faces = $\{1, 3, 5\}$ → $P(E_2) = 1/2$

$E_1 \cap E_2 = \{\phi\}$

$P(E_1 \cap E_2) = 0$ → E_1 and E_2 are mutually exclusive

$E_1 \cup E_2 = \{1, 2, 3, 4, 5, 6\}$ → $P(E_1 \cup E_2) = 1$

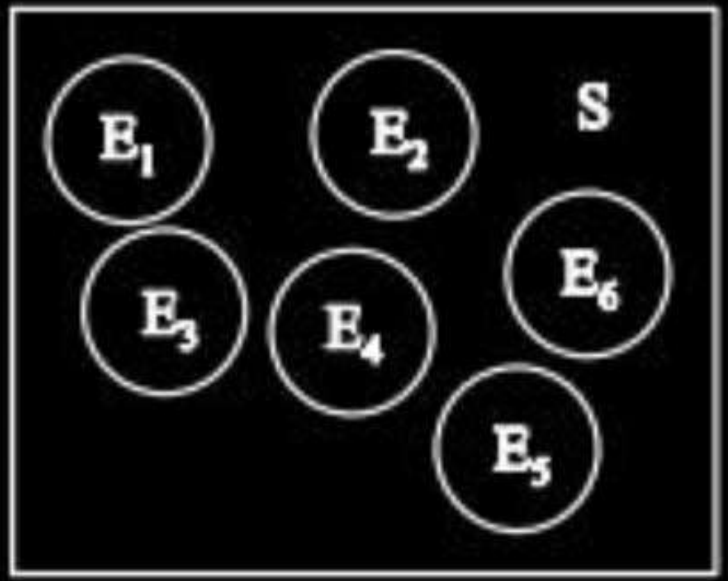
$P(E_1) + P(E_2) = 1$

Case 2:

Events E_1, E_2 .... E_n are from same sample space and they are called as mutually exclusive if :

$$E_i \cap E_j = \phi \text{ For } \forall\, i, j \quad i \neq j$$

$$P(E_i \cap E_j) = 0 \text{ For } \forall\, i, j \quad i \neq j$$



Addition theorem of probability

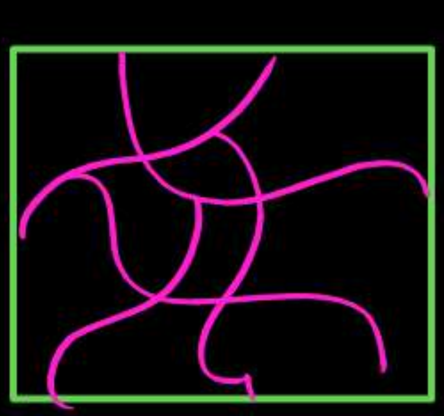
$$P(E_1 \cup E_2 \cup E_3 \ldots\ldots\ldots \cup E_n) = P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_n)$$

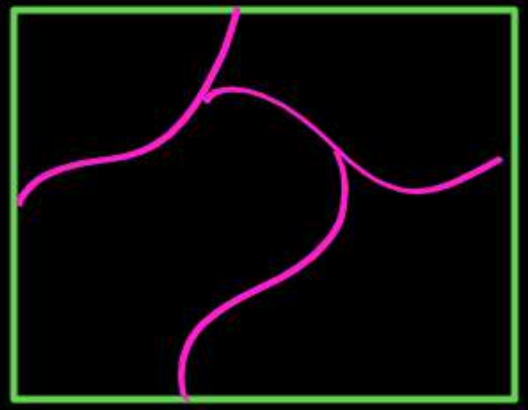
Only where $E_1, E_2 \ldots\ldots E_n$ are mutually exclusive events over same sample space

Exhaustive of events :

E_1, E_2 ………. E_n are events from same sample space

$S_1 = \{E_1, E_2$ …… $E_n\}$ exhaustive set

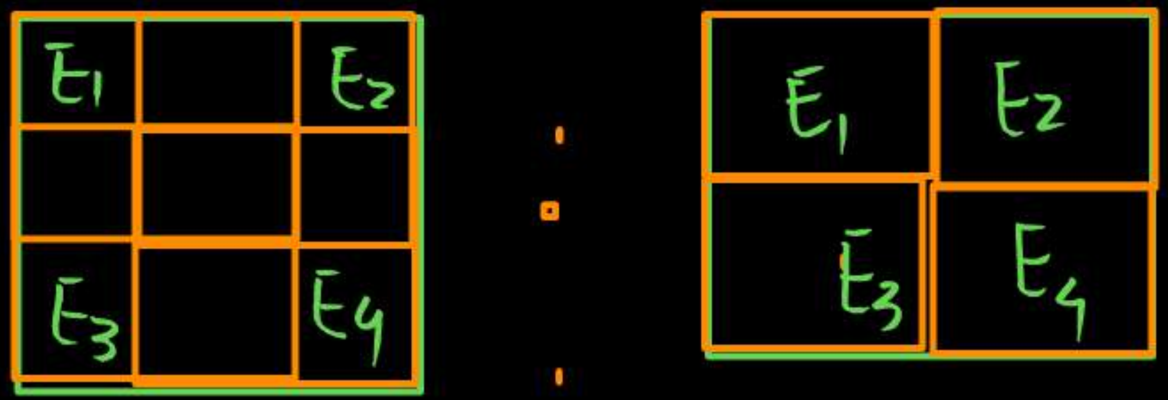


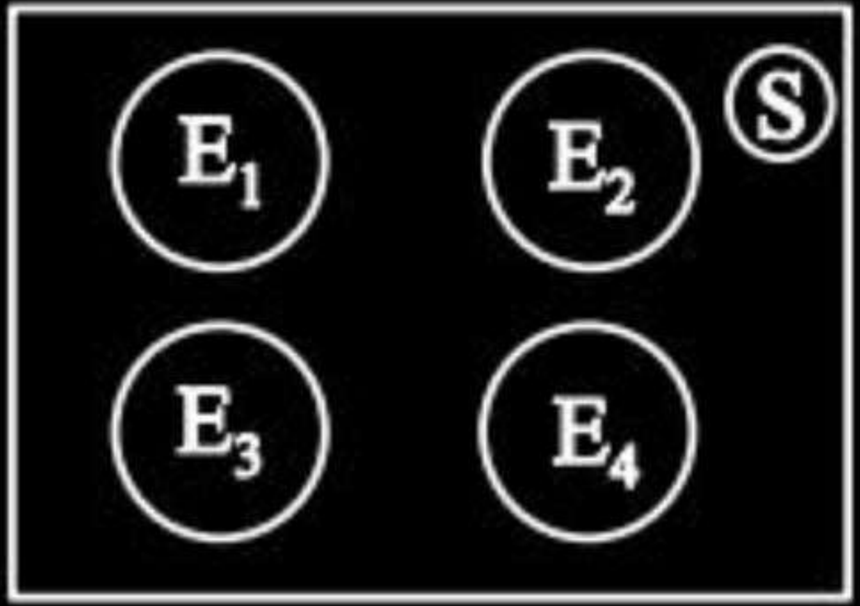


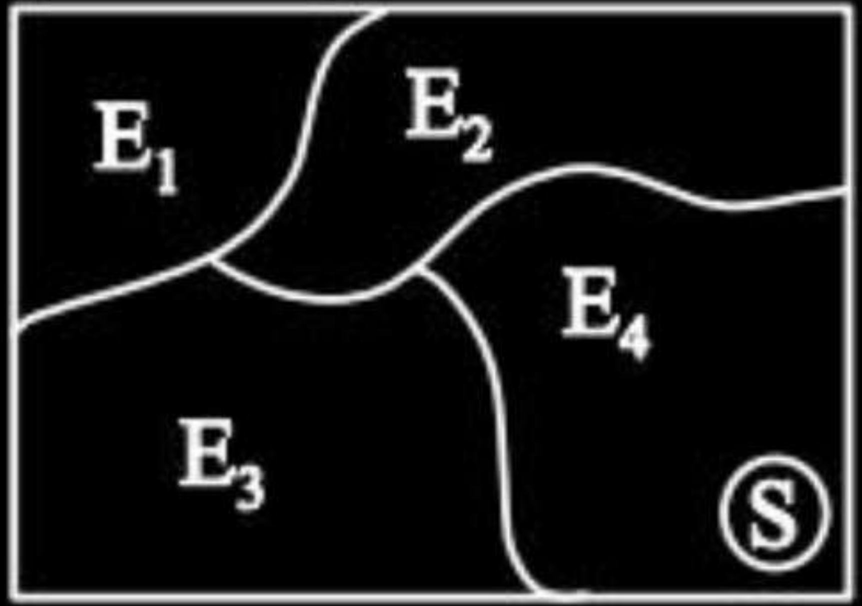
$$P(E_1 \cup E_2 \text{ ……… } \cup E_n) = P(S) = 1$$

Note:

"At least one of E_1, E_2 ………. E_n occurs when experiment is performed"







{E} = mutually exclusive

{E} = Not exhaustive

{E} = mutually exclusive

{E} = Exhaustive

Rolling of dice

$S = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{2, 4, 6\}$

$E_2 = \{1, 3, 5\}$

$E_3 = \{2, 3, 5\}$

$E_1 \cup E_2 \cup E_3 = S$

E_1, E_2, E_3 are exhaustive set of events

$P(E_1 \cup E_2 \cup E_3) = P(S) = 1$

Mutually Exclusive and Exhaustive events

$$E_1, E_2, E_3 \ldots\ldots\ldots\ldots\ldots E_n$$

$$E_1 \cup E_2 \cup E_3 \cup \ldots\ldots\ldots E_n = S$$

$$E_i \cap E_j = \phi \ \forall \ i \neq j$$

**

Independent Event :

Case 1:

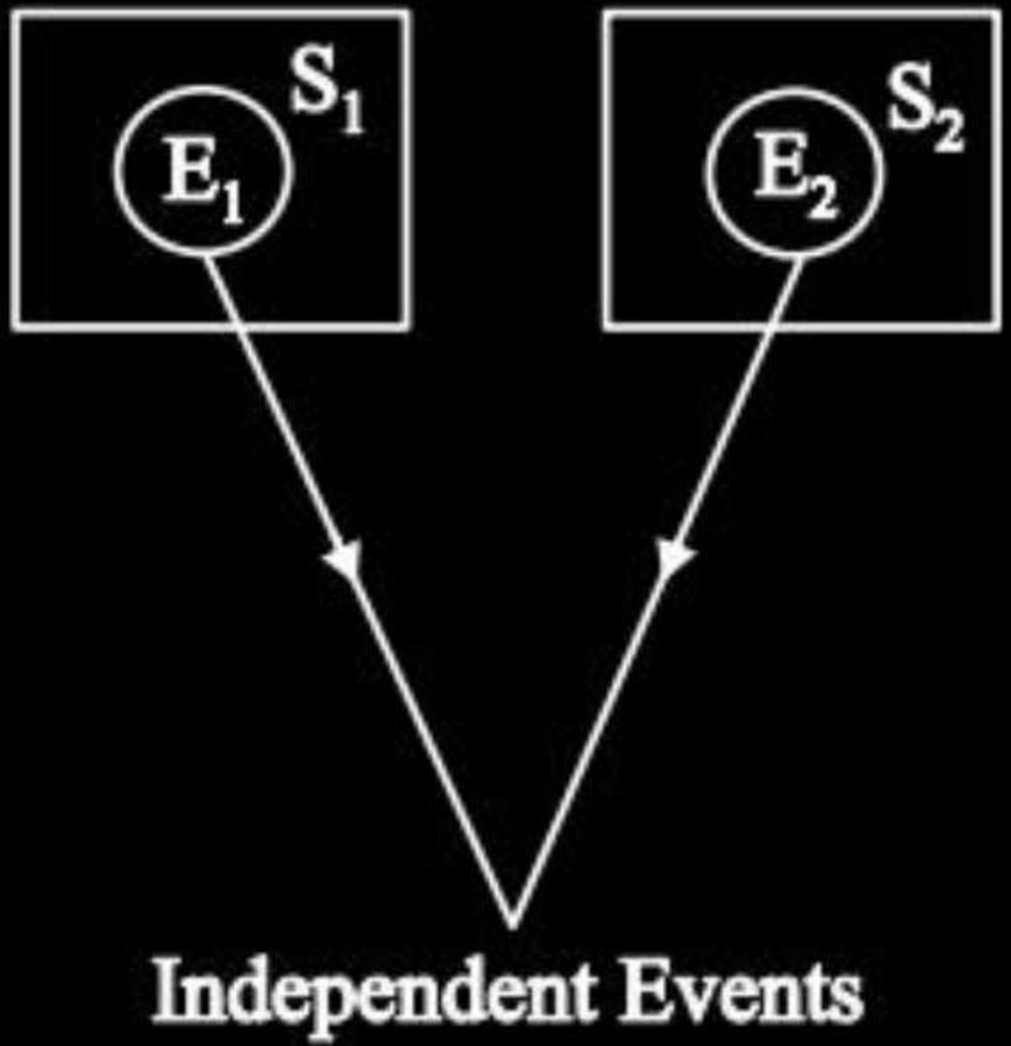
"Events A and B are said to be independent if occurrence (or non occurrence) of one event does not effect the occurrence (or nonoccurrence) of other event."

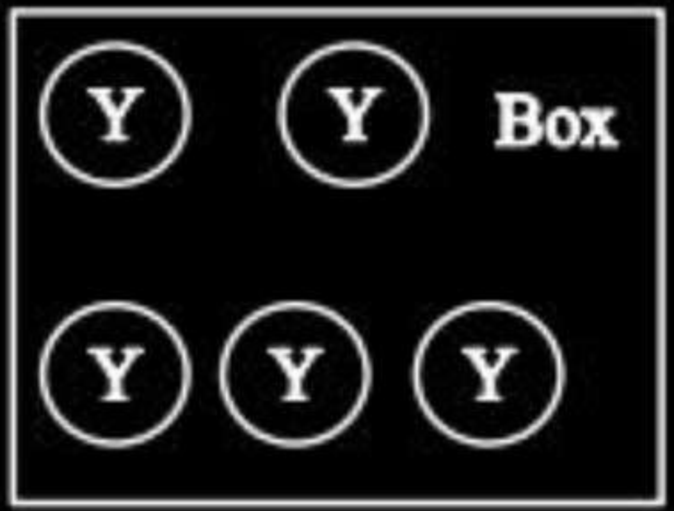
$S_1 = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{2, 4, 6\}$

$S_2 = \{H, T\}$

$E_2 = \{H\}$





E_1 and E_2 are independent event

$E_1 \rightarrow$ Pick one yellow ball

$E_2 \rightarrow$ Pick another yellow ball in next draw, replacement is allowed

If A and B are independent event

$$P\left(\frac{A}{B}\right) = P(A)$$ ✓

$$P\left(\frac{B}{A}\right) = P(B)$$ ✓

$$P(A \cap B) = P(A)P\left(\frac{B}{A}\right)$$

$$= P(B)P\left(A/B\right)$$

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cap B) = P\left(\frac{A}{B}\right)P(B) = P\left(\frac{B}{A}\right)P(A) = P(A)P(B)$$

$$P(A \cap B) = P(A)P(B)$$

Multiplication theorem

(i) A, B are two events

$$P(A \cap B) = P(A)P\left(\frac{B}{A}\right)$$

(ii) A, B, C are 3 events

$$P(A \cap B \cap C) = P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A \cap B}\right)$$

Imp

(iii) If A and B are two independent events then $(\bar{A}, B)$, $(A, \bar{B})$ $(\bar{A}, \bar{B})$ will also be independent

$P(A \cap B) = P(A)P(B)$, $P(A \cap \bar{B}) = P(A)P(\bar{B})$

(iv) If A, B C are 3 events put of which every 2 are independent

$P(A \cap B) = P(A)P(B)$, $P(B \cap C) = P(B)P(C)$,

$P(C \cap A) = P(C)P(A)$, $P(A \cap B \cap C) = ?$

(v) If A, B, C are independent events:

$P(A \cap B \cap C) = P(A)P(B)P(C)$

Multiplication theorem

E_1, E_2 …………. E_n are independent events

$$P(E_1 \cap E_2 \cap E_3 \ldots\ldots\ldots E_n) = P(E_1)\, P(E_2)\, P(E_3) \ldots\ldots\ldots\ldots P(E_n)$$

Q. Let A and B be two events such that

$P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$

Where $\bar{A}$ stands for the complement of the event A. The event A and B are

(A) Independent but not equally likely

(B) Independent and equally likely

(C) Mutually exclusion and independent

(D) Equally likely but and independent

$\overline{P(A \cup B)} = \frac{1}{6}$

$P(A \cap B) = \frac{1}{4}$

$P(\bar{A}) = \frac{1}{4}$

1.

EQUALLY LIKELY:

$P(A) = \frac{3}{4}$

$P(A \cup B) = \frac{5}{6} = P(A) + P(B) - P(A \cap B)$

$P(B) = 1/3$

2. Mutually Exclusive

$$P(A \cup B) \neq P(A) + P(B)$$

$$P(A \cap B) \neq 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

3 Independent

$$P(A \cap B) = \frac{1}{4}$$

$$P(A)\,P(B) = \frac{1}{4}$$

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

Q. Let two fair, six faced dice A and B thrown simultaneously. If E_1 is the event that dice A shows up four, E_2 is the event that dice B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which statement is not true ?

(A) E_1 and E_3 are independent

(B) E_1, E_2 and E_3 are independent

(C) E_1 and E_2 are independent

(D) E_2 and E_3 are independent

Q. Let E and F be two independent events the probability that exactly one of them occurs is 11/25 and probability of none of them occurring is 2/25. If P(T) denotes the probability of occurrence of the event T, then"

(A) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$

(B) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

(C) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$

(D) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

Q. Two coins R and S are tossed. The 4 joint events $H_R H_S$, $T_R T_S$, $H_R T_S$, $T_R H_S$ have probabilities 0.28, 0.18, 0.30, 0.24 respectively, where H represents head and T represents tail. Which one of the following is TRUE ?

GATE

(A) The coin tosses are independent

(B) R is fair, S is not

(C) S is fair, R is not

(D) The coin tosses are dependent

THANK YOU!